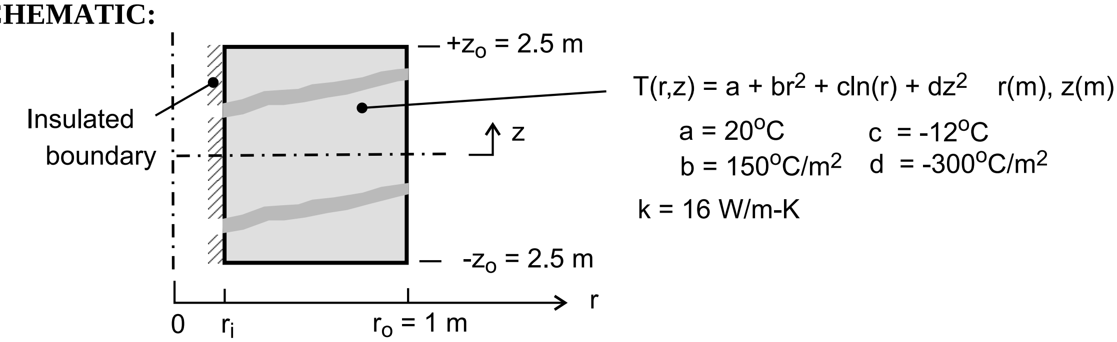
PROBLEM 2.40

KNOWN: Steady-state temperature distribution for hollow cylindrical solid with volumetric heat generation.

FIND: (a) Determine the inner radius of the cylinder, r_i, (b) Obtain an expression for the volumetric rate of heat generation, $\dot{q}$, (c) Determine the axial distribution of the heat flux at the outer surface, $q''_r(r_o, z)$, and the heat rate at this outer surface; is the heat rate *in* or *out* of the cylinder; (d) Determine the radial distribution of the heat flux at the end faces of the cylinder, $q''_z(r, +z_o)$ and $q''_z(r, -z_o)$, and the corresponding heat rates; are the heat rates *in* or *out* of the cylinder; (e) Determine the relationship of the surface heat rates to the heat generation rate; is an overall energy balance satisfied?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction with constant properties and volumetric heat generation.

ANALYSIS: (a) Since the inner boundary, r = r_i, is adiabatic, then $q''_r(r_i, z) = 0$. Hence the temperature gradient in the r-direction must be zero.

$$\left.\frac{\partial T}{\partial r}\right)_{r_i} = 0 + 2br_i + c/r_i + 0 = 0$$

$$r_i = +\left(-\frac{c}{2b}\right)^{1/2} = \left(-\frac{-12°C}{2\times 150°C/m^2}\right)^{1/2} = 0.2\ m \qquad <$$

(b) To determine $\dot{q}$, substitute the temperature distribution into the heat diffusion equation, Eq. 2.20, for two-dimensional (r,z), steady-state conduction

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \frac{\dot{q}}{k} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r[0 + 2br + c/r + 0]\right) + \frac{\partial}{\partial z}(0 + 0 + 0 + 2dz) + \frac{\dot{q}}{k} = 0$$

$$\frac{1}{r}[4br + 0] + 2d + \frac{\dot{q}}{k} = 0$$

$$\dot{q} = -k[4b - 2d] = -16\,W/m\cdot K\left[4\times 150°C/m^2 - 2\left(-300°C/m^2\right)\right]$$

$$\dot{q} = 0\,W/m^3 \qquad <$$

(c) The heat flux and the heat rate at the outer surface, r = r_o, may be calculated using Fourier's law. Note that the sign of the heat flux in the positive r-direction is negative, and hence the heat flow is *into* the cylinder.

$$q''_r(r_o, z) = -k\left.\frac{\partial T}{\partial r}\right)_{r_o} = -k\left[0 + 2br_o + c/r_o + 0\right]$$

Continued …..

$$q''_r(r_{o,}z) = -16\,\mathrm{W/m\cdot K}\left[2\times150^\circ\mathrm{C/m^2}\times1\ \mathrm{m} - 12^\circ\mathrm{C}/1\ \mathrm{m}\right] = -4608\,\mathrm{W/m^2} \qquad <$$

$$q_r(r_o) = A_r\, q''_r(r_{o,}z) \qquad \text{where} \qquad A_r = 2\pi r_o(2z_o)$$

$$q_r(r_o) = -4\pi\times1\ \mathrm{m}\times2.5\ \mathrm{m}\times4608\,\mathrm{W/m^2} = -144,765\,\mathrm{W} \qquad <$$

(d) The heat fluxes and the heat rates at end faces, $z = +z_o$ and $-z_o$, may be calculated using Fourier's law. The direction of the heat rate *in* or *out* of the end face is determined by the sign of the heat flux in the positive z-direction.

At the upper end face, $z = +z_o$: heat rate is *out* of the cylinder <

$$q''_z(r,+z_o) = -k\left.\frac{\partial T}{\partial z}\right)_{z_o} = -k\left[0+0+0+2dz_o\right]$$

$$q''_z(r,+z_o) = -16\,\mathrm{W/m\cdot K}\times2\left(-300^\circ\mathrm{C/m^2}\right)2.5\ \mathrm{m} = +24,000\,\mathrm{W/m^2} \qquad <$$

$$q_z(+z_o) = A_z\, q''_z(r,+z_o) \qquad \text{where} \qquad A_z = \pi\left(r_o^2 - r_i^2\right)$$

$$q_z(+z_o) = \pi\left(1^2 - 0.2^2\right)\mathrm{m^2}\times24,000\,\mathrm{W/m^2} = +72,382\,\mathrm{W} \qquad <$$

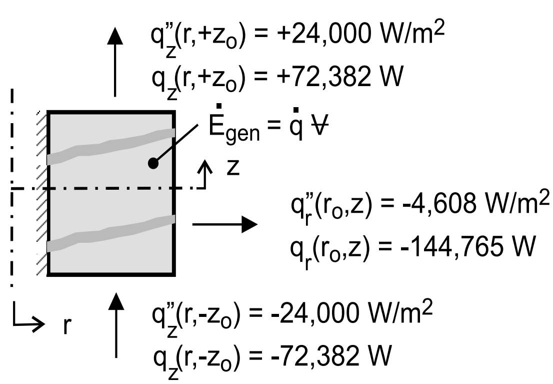
At the lower end face, $z = -z_o$: heat rate is *out* of the cylinder <

$$q''_z(r,-z_o) = -k\left.\frac{\partial T}{\partial z}\right)_{-z_o} = -k\left[0+0+0+2dz_o\right]$$

$$q''_z(r,-z_o) = -16\,\mathrm{W/m^2\cdot K}\times2\left(-300^\circ\mathrm{C/m}\right)\left(-2.5\ \mathrm{m}\right) = -24,000\,\mathrm{W/m^2} \qquad <$$

$$q_z(-z_o) = -72,382\,\mathrm{W} \qquad <$$

(e) The heat rates from the surfaces and the volumetric heat generation can be related through an overall energy balance on the cylinder as shown in the sketch.



$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0 \qquad \text{where} \qquad \dot{E}_{gen} = \dot{q}\forall = 0$$

$$\dot{E}_{in} = -q_r(r_o) = -(-144,765\,\mathrm{W}) = +144,765\,\mathrm{W} \qquad <$$

$$\dot{E}_{out} = +q_z(z_o) - q_z(-z_o) = \left[72,382 - (-72,382)\right]\mathrm{W} = +144,764\,\mathrm{W} \qquad <$$

The overall energy balance is satisfied.

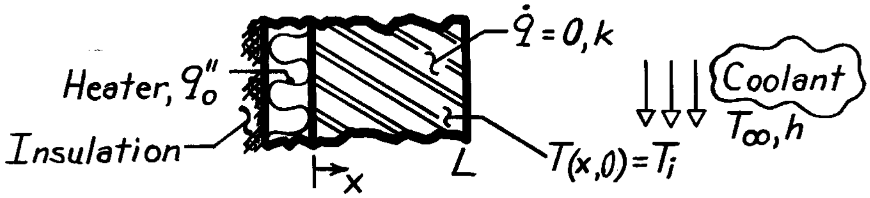
COMMENTS: When using Fourier's law, the heat flux q''_z denotes the heat flux in the positive z-direction. At a boundary, the sign of the numerical value will determine whether heat is flowing into or out of the boundary.

PROBLEM 2.47

KNOWN: Plane wall, initially at a uniform temperature T_i, is suddenly exposed to convection with a fluid at T_∞ at one surface, while the other surface is exposed to a constant heat flux q''_o.

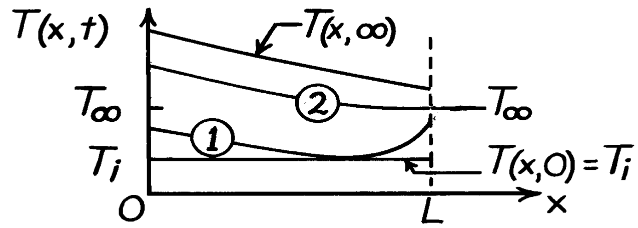
FIND: (a) Temperature distributions, T(x,t), for initial, steady-state and two intermediate times, (b) Corresponding heat fluxes on $q''_x - x$ coordinates, (c) Heat flux at locations x = 0 and x = L as a function of time, (d) Expression for the steady-state temperature of the heater, T(0,∞), in terms of q''_o, T_∞, k, h and L.

SCHEMATIC:



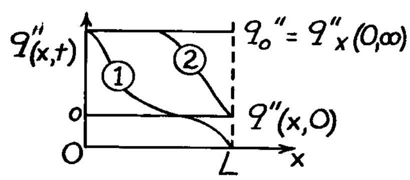
ASSUMPTIONS: (1) One-dimensional conduction, (2) No heat generation, (3) Constant properties.

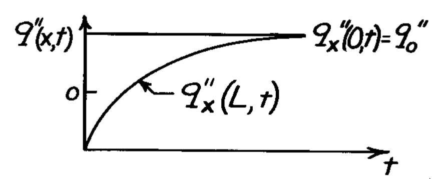
ANALYSIS: (a) For $T_i < T_\infty$, the temperature distributions are



Note the constant gradient at x = 0 since $q''_x(0) = q''_o$.

(b) The heat flux distribution, $q''_x(x,t)$, is determined from knowledge of the temperature gradients, evident from Part (a), and Fourier's law.



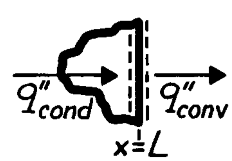


(c) On $q''_x(x,t) - t$ coordinates, the heat fluxes at the boundaries are shown above.

(d) Perform a surface energy balance at x = L and an energy balance on the wall:

$$q''_{cond} = q''_{conv} = h\left[T(L,\infty) - T_\infty\right] \quad (1), \qquad q''_{cond} = q''_o. \quad (2)$$

For the wall, under steady-state conditions, Fourier's law gives



$$q''_o = -k\frac{dT}{dx} = k\frac{T(0,\infty) - T(L,\infty)}{L}. \qquad (3)$$

Combine Eqs. (1), (2), (3) to find:

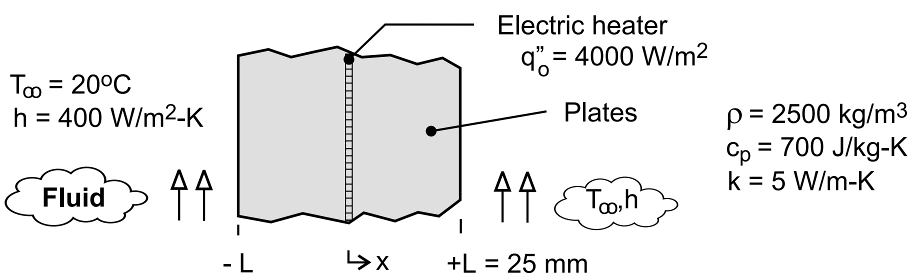
$$T(0,\infty) = T_\infty + \frac{q''_o}{1/h + L/k}.$$

PROBLEM 2.53

KNOWN: Thin electrical heater dissipating 4000 W/m^2 sandwiched between two 25-mm thick plates whose surfaces experience convection.

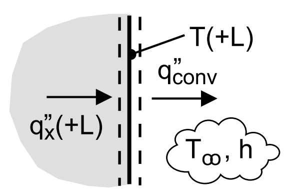
FIND: (a) On T-x coordinates, sketch the steady-state temperature distribution for $-L \le x \le +L$; calculate values for the surfaces x = L and the mid-point, x = 0; label this distribution as Case 1 and explain key features; (b) Case 2: sudden loss of coolant causing existence of adiabatic condition on the x = +L surface; sketch temperature distribution on same T-x coordinates as part (a) and calculate values for x = 0, ± L; explain key features; (c) Case 3: further loss of coolant and existence of adiabatic condition on the x = - L surface; situation goes undetected for 15 minutes at which time power to the heater is deactivated; determine the eventual ($t \rightarrow \infty$) uniform, steady-state temperature distribution; sketch temperature distribution on same T-x coordinates as parts (a,b); and (d) On T-t coordinates, sketch the temperature-time history at the plate locations x = 0, ± L during the transient period between the steady-state distributions for Case 2 and Case 3; at what location and when will the temperature in the system achieve a maximum value?

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal volumetric generation in plates, and (3) Negligible thermal resistance between the heater surfaces and the plates.

ANALYSIS: (a) Since the system is symmetrical, the heater power results in equal conduction fluxes through the plates. By applying a surface energy balance on the surface x = +L as shown in the schematic, determine the temperatures at the mid-point, x = 0, and the exposed surface, x + L.



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_x(+L) - q''_{conv} = 0 \qquad \text{where} \qquad q''_x(+L) = q''_o/2$$

$$q''_o/2 - h\left[T(+L) - T_\infty\right] = 0$$

$$T_1(+L) = q''_o/2h + T_\infty = 4000\,\mathrm{W/m^2}/\left(2 \times 400\,\mathrm{W/m^2 \cdot K}\right) + 20°\mathrm{C} = 25°\mathrm{C} \qquad <$$

From Fourier's law for the conduction flux through the plate, find T(0).

$$q''_x = q''_o/2 = k\left[T(0) - T(+L)\right]/L$$

$$T_1(0) = T_1(+L) + q''_o L/2k = 25°\mathrm{C} + 4000\,\mathrm{W/m^2 \cdot K} \times 0.025\,\mathrm{m}/\left(2 \times 5\,\mathrm{W/m \cdot K}\right) = 35°\mathrm{C} \qquad <$$

The temperature distribution is shown on the T-x coordinates below and labeled Case 1. The key features of the distribution are its symmetry about the heater plane and its linear dependence with distance.

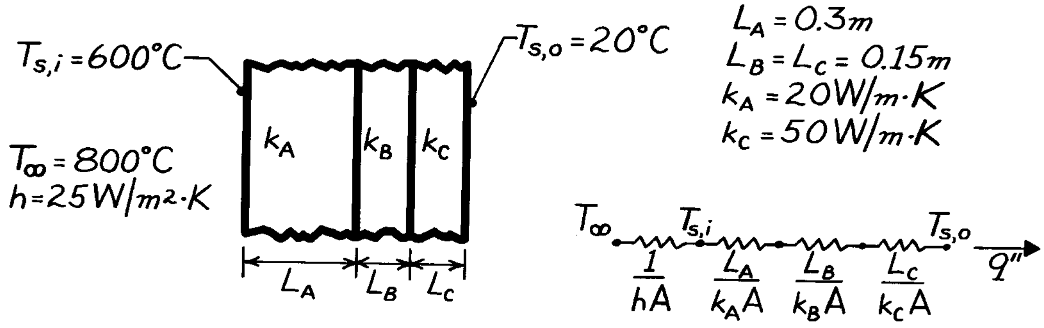
Continued …..

PROBLEM 3.9

KNOWN: Thicknesses of three materials which form a composite wall and thermal conductivities of two of the materials. Inner and outer surface temperatures of the composite; also, temperature and convection coefficient associated with adjoining gas.

FIND: Value of unknown thermal conductivity, k_B.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation effects.

ANALYSIS: Referring to the thermal circuit, the heat flux may be expressed as

$$q'' = \frac{T_{s,i} - T_{s,o}}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}} = \frac{(600 - 20)^\circ \text{C}}{\frac{0.3 \text{ m}}{20 \text{ W/m}\cdot\text{K}} + \frac{0.15 \text{ m}}{k_B} + \frac{0.15 \text{ m}}{50 \text{ W/m}\cdot\text{K}}}$$

$$q'' = \frac{580}{0.018 + 0.15/k_B} \text{ W/m}^2. \tag{1}$$

The heat flux may be obtained from

$$q'' = h\left(T_\infty - T_{s,i}\right) = 25 \text{ W/m}^2\cdot\text{K}\,(800\text{-}600)^\circ\text{C} \tag{2}$$

$$q'' = 5000 \text{ W/m}^2.$$

Substituting for the heat flux from Eq. (2) into Eq. (1), find

$$\frac{0.15}{k_B} = \frac{580}{q''} - 0.018 = \frac{580}{5000} - 0.018 = 0.098$$

$$k_B = 1.53 \text{ W/m}\cdot\text{K}. \qquad <$$

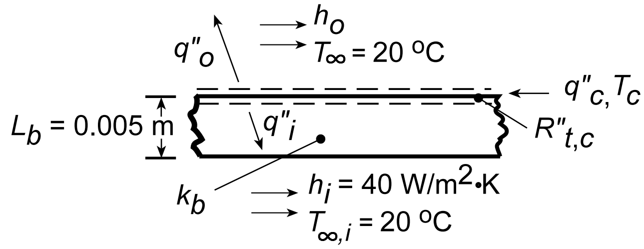
COMMENTS: Radiation effects are likely to have a significant influence on the net heat flux at the inner surface of the oven.

PROBLEM 3.27

KNOWN: Operating conditions for a board mounted chip.

FIND: (a) Equivalent thermal circuit, (b) Chip temperature, (c) Maximum allowable heat dissipation for dielectric liquid (h_o = 1000 W/m^2·K) and air (h_o = 100 W/m^2·K). Effect of changes in circuit board temperature and contact resistance.

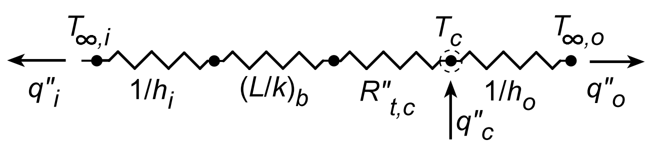
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible chip thermal resistance, (4) Negligible radiation, (5) Constant properties.

PROPERTIES: *Table A-3*, Aluminum oxide (polycrystalline, 358 K): k_b = 32.4 W/m·K.

ANALYSIS: (a)



(b) Applying conservation of energy to a control surface about the chip $\left(\dot{E}_{in} - \dot{E}_{out} = 0\right)$,

$$q''_c - q''_i - q''_o = 0$$

$$q''_c = \frac{T_c - T_{\infty,i}}{1/h_i + (L/k)_b + R''_{t,c}} + \frac{T_c - T_{\infty,o}}{1/h_o}$$

With $q''_c = 3\times10^4\ \text{W/m}^2$, h_o = 1000 W/m^2·K, k_b = 1 W/m·K and $R''_{t,c} = 10^{-4}\ \text{m}^2\cdot\text{K/W}$,

$$3\times10^4\ \text{W/m}^2 = \frac{T_c - 20^\circ\text{C}}{\left(1/40 + 0.005/1 + 10^{-4}\right)\text{m}^2\cdot\text{K/W}} + \frac{T_c - 20^\circ\text{C}}{(1/1000)\,\text{m}^2\cdot\text{K/W}}$$

$$3\times10^4\ \text{W/m}^2 = \left(33.2T_c - 664 + 1000T_c - 20{,}000\right)\text{W/m}^2\cdot\text{K}$$

$1003T_c$ = 50,664

T_c = 49°C. <

(c) For T_c = 85°C and h_o = 1000 W/m^2·K, the foregoing energy balance yields

$$q''_c = 67{,}160\ \text{W/m}^2$$ <

with q''_o = 65,000 W/m^2 and q''_i = 2160 W/m^2. Replacing the dielectric with air (h_o = 100 W/m^2·K), the following results are obtained for different combinations of k_b and $R''_{t,c}$.

Continued...

PROBLEM 3.27 (Cont.)

k_b (W/m·K)	$R''_{t,c}$ (m^2·K/W)	q''_i (W/m^2)	q''_o (W/m^2)	q''_c (W/m^2)
1	10^{-4}	2159	6500	8659
32.4	10^{-4}	2574	6500	9074
1	10^{-5}	2166	6500	8666
32.4	10^{-5}	2583	6500	9083

<

COMMENTS: 1. For the conditions of part (b), the total internal resistance is 0.0301 m^2·K/W, while the outer resistance is 0.001 m^2·K/W. Hence

$$\frac{q''_o}{q''_i} = \frac{\left(T_c - T_{\infty,o}\right)/R''_o}{\left(T_c - T_{\infty,i}\right)/R''_i} = \frac{0.0301}{0.001} = 30 .$$

and only approximately 3% of the heat is dissipated through the board.

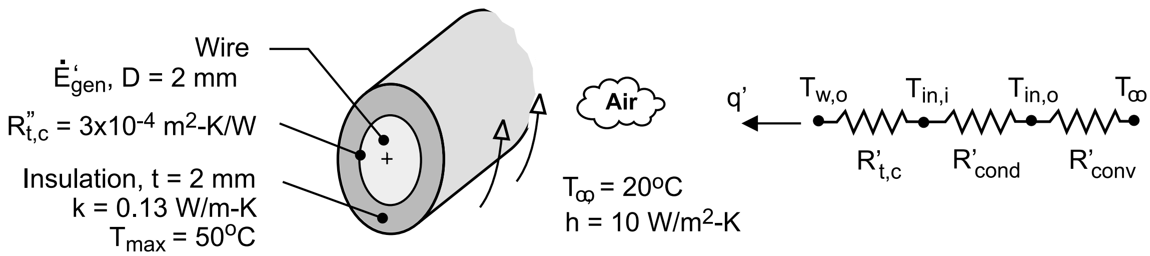
2. With h_o = 100 W/m^2·K, the outer resistance increases to 0.01 m^2·K/W, in which case $q''_o / q''_i = R''_i / R''_o$ = 0.0301/0.01 = 3.1 and now almost 25% of the heat is dissipated through the board. Hence, although measures to reduce R''_i would have a negligible effect on q''_c for the liquid coolant, some improvement may be gained for air-cooled conditions. As shown in the table of part (b), use of an aluminum oxide board increase q''_i by 19% (from 2159 to 2574 W/m^2) by reducing R''_i from 0.0301 to 0.0253 m^2·K/W. Because the initial contact resistance ($R''_{t,c} = 10^{-4} m^2 \cdot K/W$) is already much less than R''_i, any reduction in its value would have a negligible effect on q''_i. The largest gain would be realized by increasing h_i, since the inside convection resistance makes the dominant contribution to the total internal resistance.

PROBLEM 3.43

KNOWN: Diameter of electrical wire. Thickness and thermal conductivity of rubberized sheath. Contact resistance between sheath and wire. Convection coefficient and ambient air temperature. Maximum allowable sheath temperature.

FIND: Maximum allowable power dissipation per unit length of wire. Critical radius of insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction through insulation, (3) Constant properties, (4) Negligible radiation exchange with surroundings.

ANALYSIS: The maximum insulation temperature corresponds to its inner surface and is independent of the contact resistance. From the thermal circuit, we may write

$$\dot{E}'_g = q' = \frac{T_{in,i} - T_\infty}{R'_{cond} + R'_{conv}} = \frac{T_{in,i} - T_\infty}{\left[\ell n\left(r_{in,o}/r_{in,i}\right)/2\pi k\right] + \left(1/2\pi r_{in,o} h\right)}$$

where $r_{in,i} = D/2 = 0.001\text{m}$, $r_{in,o} = r_{in,i} + t = 0.003\text{m}$, and $T_{in,i} = T_{max} = 50°\text{C}$ yields the maximum allowable power dissipation. Hence,

$$\dot{E}'_{g,max} = \frac{(50-20)°\text{C}}{\dfrac{\ell n\, 3}{2\pi \times 0.13\,\text{W/m}\cdot\text{K}} + \dfrac{1}{2\pi(0.003\text{m})10\,\text{W/m}^2\cdot\text{K}}} = \frac{30°\text{C}}{(1.35+5.31)\,\text{m}\cdot\text{K/W}} = 4.51\,\text{W/m} \qquad <$$

The critical insulation radius is also unaffected by the contact resistance and is given by

$$r_{cr} = \frac{k}{h} = \frac{0.13\,\text{W/m}\cdot\text{K}}{10\,\text{W/m}^2\cdot\text{K}} = 0.013\text{m} = 13\,\text{mm} \qquad <$$

Hence, $r_{in,o} < r_{cr}$ and $\dot{E}'_{g,max}$ could be increased by increasing $r_{in,o}$ up to a value of 13 mm (t = 12 mm).

COMMENTS: The contact resistance affects the temperature of the wire, and for $q' = \dot{E}'_{g,max} = 4.51\,\text{W/m}$, the outer surface temperature of the wire is $T_{w,o} = T_{in,i} + q' R'_{t,c} = 50°\text{C} + (4.51\,\text{W/m})\left(3\times10^{-4}\,\text{m}^2\cdot\text{K/W}\right)/\pi(0.002\text{m}) = 50.2°\text{C}$. Hence, the temperature change across the contact resistance is negligible.